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Power law decay in model predictability skill

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[1] Ocean predictability skill is investigated using a Gulf of Mexico nowcast/forecast model. Power law scaling is found in the mean square error of displacement between drifting buoy and model trajectories (both at 50 m depth). The probability density function of the model valid prediction period (VPP) is asymmetric with a long and broad tail on the higher value side, which suggests long-term predictability. The calculations demonstrate that the long-term (extreme long such as 50–60 day) predictability is not an "outlier" and shares the same statistical properties as the short-term predictions. *INDEX TERMS:* 4263 Oceanography: General: Ocean prediction; 3210 Mathematical Geophysics: Modeling; 4532 Oceanography: Physical: General circulation; 4512 Oceanography: Physical: Currents

1. Introduction

[2] Two kinds of predictability exist in ocean (and atmospheric) models. The predictability of the first kind (second) describes the model performance due to imprecise initial conditions (boundary conditions or model parameters) [*Lorenz*, 1984]. For a perfect model, a universal exponential law is found in the first type predictability for prediction error growth during the initial stage (linear regime) due to infinitesimal and small initial errors [*Lorenz*, 1984; *Robinson*, 1996].

[3] Neither the model dynamics nor the model forcing is identical to that of the real ocean. For example, the lateral transport through straits and fluxes between the atmosphere and ocean are not precisely known apriori. Uncertain model parameters and unresolved spatial and temporal scales distort the dynamical equations (the evolution law). The forecast sensitivity to such distortions is referred to as the predictability of the second kind [*Lorenz*, 1984]. *Chu* [1999] has investigated the two kinds of predictability using the Lorenz attractor.

[4] On the base of the first-passage time (i.e., the time period for a moving particle inside a domain to first impacts its boundary), *Chu et al.* [2002a, 2002b] proposed that the valid prediction period (VPP) effectively represents predictability. VPP is defined as the time period when the prediction error first exceeds a pre-determined criterion ε (i.e., the tolerance level). The conditional probability density function of VPP with a given initial error satisfies the backward Fokker-Planck equation (called Pontryagin-Kolomogorov equation in Russian literature). Using VPP as a quantitative measure for prediction skill, both linear and nonlinear regimes of forecast errors were found in the low-order atmospheric Lorenz model [*Lorenz*, 1984]. In the linear regime (usually for the predictability of the first kind during the initial stage), the error growth follows the exponential law (linear scaling law) with VPP represented by the e-folding time. In the nonlinear regime, the error growth does not follow the exponential law, and VPP cannot be represented by the e-folding time.

[5] The existence of the power law (nonlinear scaling law) is well known in oceanic and atmospheric transport processes [*Osborne et al.*, 1989; *Meyers et al.*, 1994; *Kyong-Hwan and Bowman*, 2000]. Questions arise: Does the power law exist in the error growth? Where does it occur? Since the exponential law was found in the predictability of the first kind, it is reasonable to check if the power law of error growth exists.

[6] The goal of this study is to demonstrate the existence of the power law of error growth using a Gulf of Mexico nowcast/forecast model and the drifter buoy data and to find a long-term (50–60 day) predictability. The predictability skill is found to have a long tail (on the high value side) following the power decay law ($t^{-\lambda}$) for long timescales, where Δ is the scaling exponent. It is well-known [*Karney*, 1983] that such a behavior leads to a slower error increase than the exponential growth. Thus, the error growth following the power law can be referred to as the long-term predictability.

2. Quantitative Measures of Model Predictability Skill

[7] Let $\mathbf{y}(t) = [y_1(\mathbf{x}_1, t), y_2(\mathbf{x}_2, t), \ldots, y_N(\mathbf{x}_N, t)]$ be a set of trajectories from N drifter buoys, and $\mathbf{y}^{(m)}(t) = [y_1^{(m)}(\mathbf{p}_1, t), y_2^{(m)}(\mathbf{p}_2, t), \ldots, y_N^{(m)}(\mathbf{p}_N, t)]$ be a set of model trajectories from N synthetic particles. Here $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$ and $(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N)$ denote the initial positions of drifter buoys and synthetic particles, respectively. Thus, the forecast error is represented by

$$\mathbf{z}(t) = \mathbf{y}^{(m)}(\mathbf{t}) - \mathbf{y}(\mathbf{t}),$$

$$\mathbf{z}(t_0) = \mathbf{z}_0 = [\mathbf{p}_1 - \mathbf{x}_1, \mathbf{p}_2 - \mathbf{x}_2, \ldots, \mathbf{p}_N - \mathbf{x}_N].$$

The error mean and variance are computed by

$$L_1 = \langle \mathbf{z} \rangle, L_2 = \left\langle (\mathbf{z} - \langle \mathbf{z} \rangle)^t (\mathbf{z} - \langle \mathbf{z} \rangle) \right\rangle \tag{1}$$

where the bracket indicates the ensemble averaging. If the error growth follows an exponential law

$$L_1 \propto e^{\sigma t}, \qquad L_2 \propto e^{\omega t}, \tag{2}$$

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σ and ω are the growth rate, and if the error growth is described by a power law

$$L_1 \propto t^{\alpha}, \qquad L_2 \propto t^{\beta}, \tag{3}$$

α and β are scaling exponents.

[8] The power law (3) can be interpreted in terms of anomalous transport and diffusion [*Weeks et al.*, 1996]. If the mean displacement grows linearly with time ($\alpha = 1$), there is a well-defined transport velocity. The condition $\alpha \neq 1$ is referred to as anomalous advection, and the condition $\beta \neq 1$ is referred to as anomalous diffusion with $0 < \beta < 1$ as sub-diffusion, $1 < \beta < 2$ as supper-diffusion, and $\beta = 2$ as ballistic dispersion occurring in the shear flows.

[9] VPP is represented by a time period τ at which $|\mathbf{z}|$ first exceeds a predetermined tolerance level ε. The probability density function (PDF) of VPP, $W(t, \mathbf{z}_0, \varepsilon)$, with the initial error $\mathbf{z}_0$ and tolerance level ε, satisfies the backward Fokker-Planck equation [*Chu et al.*, 2002a, 2002b; *Chu and Ivanov*, 2002]. The predictability skill can also be verified using the probability density function of prediction (PDP), which is defined by successful prediction for the period, $t - t_0$ [*Ivanov et al.*, 1994; *Ivanov and Margolina*, 1999],

$$\begin{aligned} \mathrm{PDP}(\tau \geq t - t_0, \mathbf{z}_0, \varepsilon) &= P(t_0, \mathbf{z}_0, \varepsilon, t - t_0) \\ &= 1 - \int_0^{t-t_0} W(\tau, \mathbf{z}_0, \varepsilon) d\tau, \end{aligned} \tag{4}$$

where t_0 is the initial time. If a power law (3) exists for the prediction errors, PDP should also follow the power law,

$$P(t_0, \mathbf{z}_0, \varepsilon, t - t_0) \sim t^{-\gamma} \quad \text{for large } t. \tag{5}$$

3. The Gulf of Mexico Nowcast/Forecast System and Drifter Data

[10] The Gulf of Mexico real time nowcast/forecast system is taken as an example to show the existence of the power decay law and the range of the three power parameters (α, β, γ,) in the predictability skill. This nowcast/forecast system [*Kantha et al.*, 1999; *Schaudt et al.*, 2001; *Kantha and Clayson*, 2000] is based on the University of Colorado version of the Princeton Ocean Model (POM) with 1/12° horizontal resolution. Real-time altimetric sea surface height (SSH) anomalies derived from NASA/CNES TOPEX and ESA ERS-1/2 altimeters, and composite MCSST data derived from NOAA AVHRR assimilated into the model in a continuous data assimilation mode are used to produce a nowcast and a four-week forecast. *Kantha and Clayson* [2000] found that forecast retains considerable skill to about 1–2 weeks, beyond which the forecast begins to deviate from reality.

[11] The Gulf of Mexico velocity data at 50 m depth were archived every six hours from the nowcast/forecast system for six months beginning on July 9, 1998. The observational data were obtained for 50 satellite-tracked drift buoys drouged at 50 m depth. These buoys were manufactured by the Applied Technology Associates, deployed by the Horizon Marine Inc., and tracked by the Louisiana State University's Coastal Studies Institute. Navigation errors for the buoy position are less than 100 m.

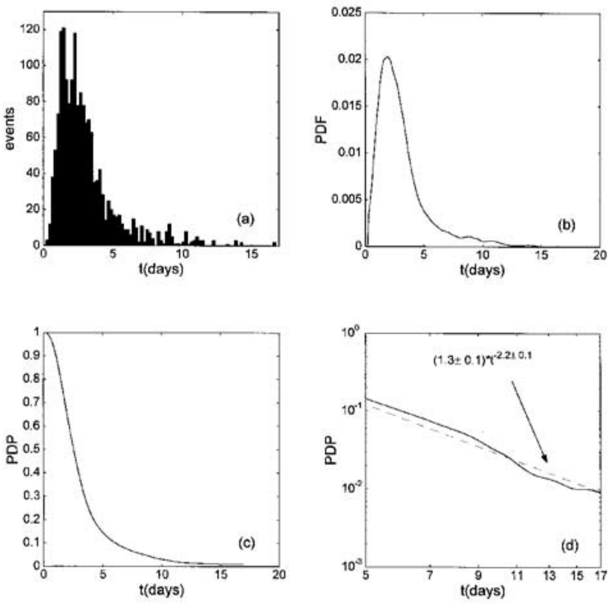

Figure 1. Statistical characteristics of VPP for zero initial error and 55 km tolerance level: (a) histogram of VPP, (b) PDF of VPP computed using the Epanechnikov kernel density, (c) PDP for VPP between 0 to 20 days, and (d) PDP for VPP between 5 to 17 days.

4. Power Law of Error Growth in Non-Ensemble Prediction

[12] Since the number of drift buoys is limited (50 in this study), the point stochastic process is used to create more realizations to calculate the ensemble mean of the predictability skill [*Tikhnov and Khimenko*, 1998]. Moving along the drifter trajectory $\mathbf{y}(t)$, points are randomly selected and the predictability skill is calculated for each of these points. Such a process is called the point stochastic process. An average over these points is equivalent to the ensemble average due to the ergodic feature of the trajectories [*Dymnikov and Filatov*, 1977; *Tikhonov and Khimenko*, 1998].

[13] VPP is computed for non-ensemble prediction with tolerance level (ε) of 55 km and without initial error ($\mathbf{z}_0 = 0$). Its PDF, $W(t, 0, 55\ \mathrm{km})$ (Figure 1b), calculated using the Epanechnikov kernel density [*Good*, 1996] from the histogram (Figure 1a), shows a non-Gaussian distribution with a narrow peak and a long tail in the domain of long-term prediction. Successful 10–15 day predictions (Figure 1c) are abnormally long (called extreme long predictions) compared to the mean VPP (3.2 days) and the most probable VPP (2.4 day). The tail of PDP follows the power law (Equation 5) with the power exponent, $\gamma = 2.2$ (Figure 1d).

[14] The error mean (L_1) and variance (L_2) are calculated at 1320 points generated using the stochastic point process. The error growth from July 9, 1998 (day-1) to August 3, 1998 (day-25) shows the existence of the

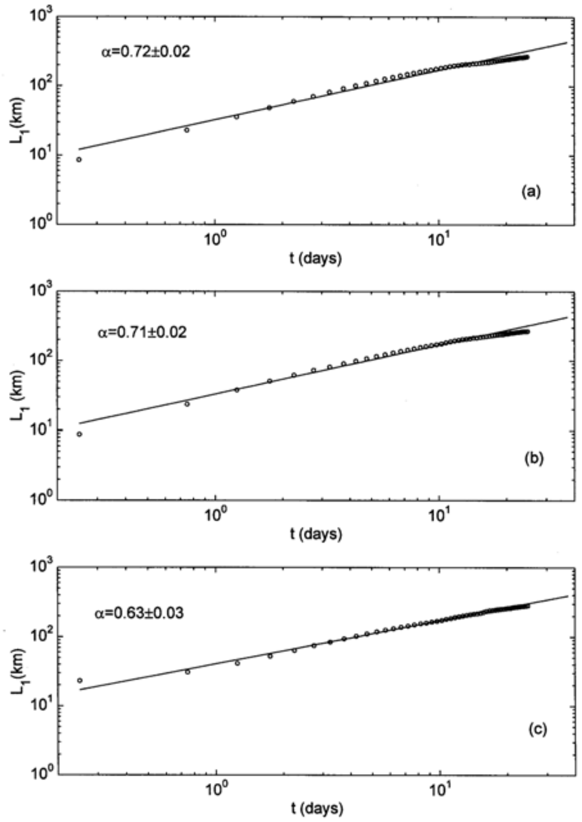

Figure 2. Scaling behavior of the mean error (L_1) growth for initial error level of (a) zero, (b) 2.2 km, and (c) 22 km, respectively.

power laws in L_1 and L_2 with the scaling exponents $\alpha = 0.72 \pm 0.02$ (Figure 2a) and $\beta = 1.34 \pm 0.02$ (Figure 3a), respectively.

5. Power Law of Error Growth in Ensemble Prediction

[15] For ensemble prediction, the initial position of synthetic particles is perturbed from the initial position of drifter buoys

$$\mathbf{p}_k = \mathbf{x}_k + \mathbf{r}_k, \tag{6}$$

where $\mathbf{r}_k$ is the initial perturbation and its magnitude varies from 2.2 to 22 km. The error mean (L_1) and variance (L_2), obtained from the ensemble prediction, also follow the power law. For example, the power for L_1 varies from α = 0.71 ± 0.02 for initial error level of 2.2 km (Figure 2b), to α = 0.63 ± 0.03 for initial error level of 22.5 km (Figure 2c). The power for L_2 varies from β = 1.4 ± 0.02 for initial error level of 2.2 km (Figure 3b), to β = 1.3 ± 0.02 for initial error level of 22.5 km (Figure 3c). Ensemble prediction (uncertainty inserted in initial condition) does not distort the power law. It only changes the value of scaling exponents α and β. Thus, the forecast errors for the predictability of the second kind (following a power law) grow considerably slower than for the predictability of the first kind (following an exponential law).

6. Long Term Predictability With Various Tolerance Levels

[16] To understand the effect of tolerance level on the long-term predictability (or the power law), the PDF of VPP is computed with zero initial error and with varying ε (Figure 4). All the PDFs show a similar pattern to PDF for ε = 55 km (Figure 1b): a non-Gaussian distribution with a narrow peak and a long tail in the domain of long-term prediction. The larger the tolerance level (ε), the longer the tail. Variation of tolerance level does not distort the power law and the long-term prediction. Extreme long-term prediction exists even for short mean and most probable VPP. For example, the VPP of extreme long-term prediction can reach 50–60 days for $\varepsilon \geq 165$ km when the mean VPP is within 10–15 days (Figure 4b). Moreover, the extreme long-term predictions are not "outliers" [*L'vov et al.*, 2001], since the same PDF is applied to short- and long-term predictions.

7. Statistical Similarity

[17] The physical reason for the existence of extreme long-term prediction is the statistical similarity between modeled and observed circulation patterns. To understand

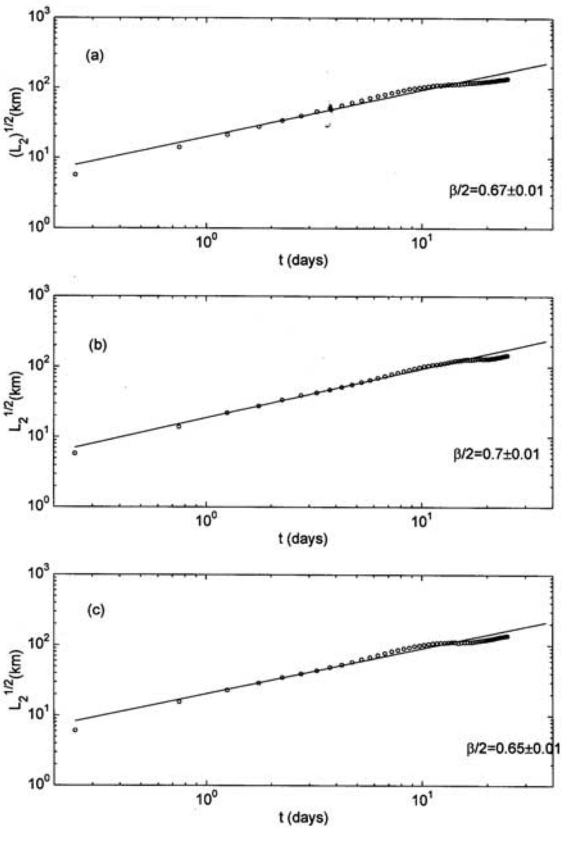

Figure 3. Scaling behavior of the error variance (L_2) growth for initial error level of (a) zero, (b) 2.2 km, and (c) 22 km, respectively.

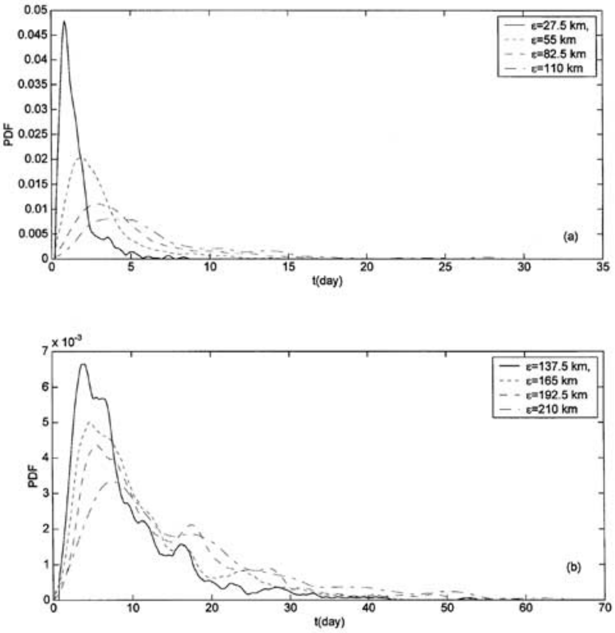

Figure 4. PDF of VPP calculated with different tolerance levels: (a) from 27.5 to 110 km, and (b) from 137.5 to 210 km.

such a similarity, empirical orthogonal function (EOF) analysis is conducted for the Gulf of Mexico circulation. The EOFs $\{\Psi_i\}$ and the corresponding amplitudes $\{A_i(t)\}$ for the streamfunction are computed from the model velocity fields at 50 m depth during the six month period (April 10–October 10, 1998) using the method proposed by *Penenko and Protasov* [1978]. The first ten EOFs consist of more than 90% of the total variability.

[18] The velocity along the drift trajectories is represented by

$$\mathbf{V} = \sum_{i=1}^{10} B_i(t)\mathbf{k} \cdot \nabla\Psi_i \tag{7}$$

where **k** is the unit vector in the vertical (upward positive), and ∇ is the horizontal gradient operator. The mode amplitudes $\{B_1(t), \ldots, B_{10}(t)\}$ are estimated from the drifter buoy data [*Eremeev et al.*, 1992]. The sporadic velocity vector field of the drifter buoys at 00:00 on July 9, 1998 (Figure 5) is reconstructed into a gridded velocity vector field (Figure 5b) using Equation (7). The modeled velocity vector field (Figure 6a) is consistent with the reconstructed velocity vector field (Figure 6b). It is noticed that the domain in Figures 5 and 6 is at 50 m depth.

[19] The two curves $\{A_i(\mathrm{t})\}$ and $\{B_i(\mathrm{t})\}$ in any amplitude pair (i = 1, 2,. . ., 10) have seven crossing points within the 6 month period. At the crossing points, there is no difference between the modeled and reconstructed (from drift buoy data) velocity fields. The Kolmogorov-Smirnov probabilities for $\{A_1, B_1\}$, $\{A_2, B_2\}, \ldots, \{A_{10}, B_{10}\}$ are, 0.63 ± 0.4, 0.79 ± 0.3,. . ., 0.39 ± 0.2 respectively. This confirms the statistical similarity between $\{A_i(\mathrm{t})\}$ and $\{B_i(\mathrm{t})\}$.

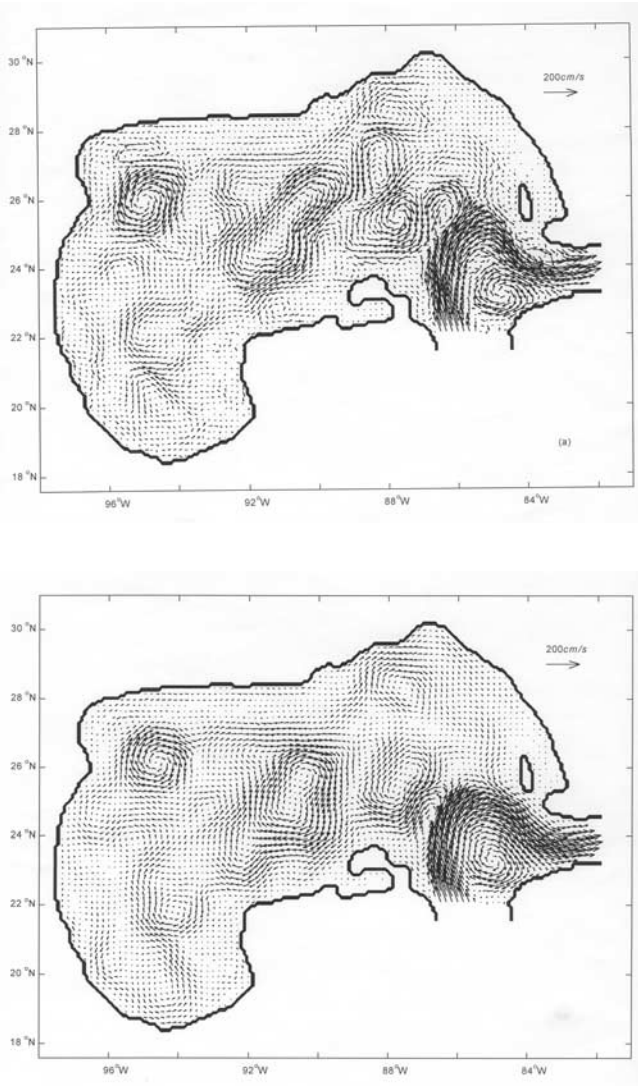

Figure 6. Velocity vectors on 00:00 July 9, 1998 obtained using (a) the Gulf of Mexico model and (b) the reconstruction scheme on the drift data.

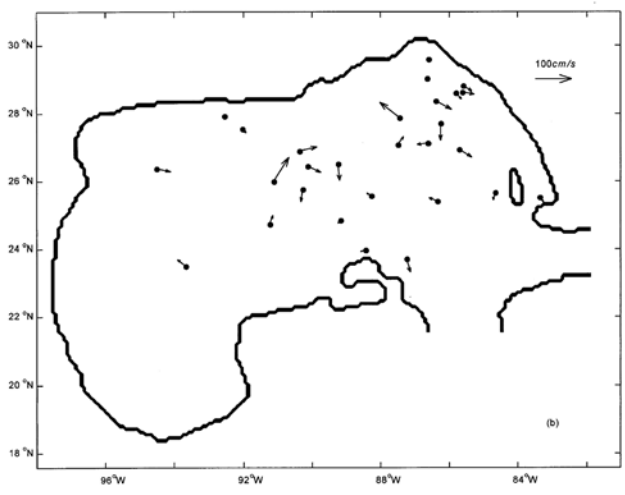

Figure 5. Velocity vectors of drifters calculated on 00:00 July 9, 1998 at 50 m depth.

8. Conclusions

[20] A power law exists in ocean model error growth. The probability density function of VPP for the Gulf of Mexico nowcast/forecast system is asymmetric with a long and broad tail on the higher value side, which indicates long-term (50–60 day) predictability. Such a long tail corresponds to a power law for the error growth. Individual model predictions can be valid for a long period (long-term prediction), even for short mean and the most probable VPP (10–15 day). The extreme long and short predictions share the same statistics.

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